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ABSTRACT

High gain and large bandwidth of traveling-wave parametric amplifier exploiting the nonlinearity of Josephson Junctions can be achieved by fulfilling the so-called phase-matching condition. This condition is usually addressed by placing resonant structures along the waveguide or by periodic modulations of its parameters, creating gaps in the waveguide's dispersion. Here, we propose to employ the Josephson junctions, which constitute the centerline of the amplifier, as resonant elements for phase matching. By numerical simulations in JoSIM (and WRspice) software, we show that Josephson plasma oscillations can be utilized to create wave vector mismatch sufficient for phase matching as well as to prevent the conversion of the pump energy to higher harmonics. The proposed traveling wave parametric amplifier design has a gain of 15 dB and a 3 GHz bandwidth, which is comparable to the state-of-the-art TWPAs.

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I. INTRODUCTION

The Josephson traveling wave parametric amplifier (JTWPA) is a superconducting device that exploits the nonlinearity of Josephson junctions embedded in a transmission line (TL) to amplify weak microwave signals with minimal added noise. The principle of operation is based on the interaction of the signal wave co-propagating with a strong driving wave, called the pump, via a nonlinear mixing process known as parametric amplification.^{1–4} JTWPAs, as a practically lossless medium,⁵ can amplify signals with noise performance approaching the quantum limit, making them ideal for quantum computing and sensitive measurement applications.^{5,6} Among the biggest challenges in TWPAs development are the mitigation of unwanted nonlinear processes, like nonlinear phase shifts and the higher-harmonic generation.⁷ The nonlinear phase shift arises as the strong pump propagates at a higher phase velocity, which leads to the deterioration of the amplifier's performance. Conversely, the generated higher harmonics cause a leakage of the pump power and its depletion.⁸

These challenges were addressed by so-called dispersion engineering—adjusting the dispersion relation of the TL to compensate the nonlinear phase shift of the pump and to avoid the higher-harmonic generation. Several designs of JTWPAs have been

proposed and studied: Photonic-Crystal design,⁹ Coupled Asymmetric SQUIDs chain,^{10,11} SQUID Chain Design,^{8,12–18} Resonator Phase Matching,^{6,15,19–22} and Bi-SQUID Based Design.²³

In this paper, we propose a simple design of JTWPA, enabling phase matching and preventing the generations of higher harmonics.

We have chosen the 3WM nonlinear processes for amplification, since they have distinct advantages relative to four-wave mixing (4WM),^{8,12,24} namely, lower pump power, reduced noise, broader bandwidth, and more stable operation. These benefits make 3WM particularly suitable for applications in quantum information processing and in sensitive, low-noise amplification. Although 4WM offers advantages such as higher potential gain and more flexible frequency conversion, our 3WM JTWPA, with plasma oscillation phase matching, provides a high 20 dB gain, which is sufficient for preamplifying signals for commercial HEMT amplifiers.

First, we provide a straightforward method to analyze transmission lines with resonant elements and their utilization in dispersion engineering. Then, we describe a metawaveguide where the Josephson junctions provide both the nonlinear inductance and the resonant characteristics at the junction's plasma frequency.

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The plasma frequency is determined by the junction's own capacitance and by an additional, designed one. The feasibility of this design, as estimated from basic TWPA theory, is confirmed by numerical simulation of a TL consisting of 2000 JJs. The results of the simulations provide further insight into the influence of plasma oscillations on nonlinear processes as well as harmonics generation and pump depletion.

Numerical calculations enable us to consider higher harmonics⁸ and other nonlinear processes beyond the reach of analytical methods. For simulations, we utilized recently developed software, JoSIM,^{25,26} which employs modern numerical methods to analyze Josephson junction circuits. We compare the performance of this software with the most renowned one, WRspice,^{27,28} which is widely used in the design and analysis of numerous amplifiers.^{8,14,15,17,21,22} We verified that our device performs excellently in both softwares; although the magnitude of the gain and the ripples are slightly different, the overall shape of the gain profile is the same.

II. CIRCUIT DESCRIPTION

A. Gain of the amplifier

The processes in TWPAs are commonly explained within the coupled mode equations (CME) describing mixing between the pump, signal, and idler with respective angular frequencies $\omega_p, \omega_s, \omega_i$ satisfying $\omega_p = \omega_s + \omega_i$. In general, wave vectors corresponding to these modes exhibit dispersion, leading to the difference in the wave vectors $\Delta k = k_p - k_s - k_i$. The CME result for the gain of TWPA with length L under the three-wave-mixing (3WM) is

$$G(L) = \cosh^2(gL) + \left(\frac{\beta}{2g}\right)^2 \sinh^2(gL), \quad (1)$$

with the gain per unit length

$$g = \sqrt{\frac{k_s k_i}{4} \left(\frac{I_p I_d}{I_d^2 + I_s^2} \right)^2 - \frac{\beta^2}{4}}, \quad (2)$$

and the effective nonlinear phase shift per unit length

$$\beta = \Delta k \left(1 + \frac{I_p^2}{4(I_d^2 + I_s^2)} \right) - \frac{I_p^2}{8(I_d^2 + I_s^2)} k_p. \quad (3)$$

Here, $I_s = \sqrt{2}I_c$, where I_c is the critical current of Josephson junction and I_p and I_d are the amplitudes of the pump and DC current, respectively. Adjusting the dispersion $k(\omega)$ to minimize β leads to higher gain. Moreover, $\beta = 0$ results in a much larger bandwidth and maximal gain, which scales exponentially with the TWPA's length, in contrast to the quadratic gain obtained from linear dispersion.

B. Dispersion relation

To estimate the dispersion relation $k(\omega)$ in a transmission line with various resonant elements, the standard TL model²⁹ is

modified [Fig. 1(b)]. The original series inductance and ground capacitance are replaced with a general series impedance $Z_S(\omega)dz$ and ground admittance $Y_G(\omega)dz$ in harmonic representation at frequency ω [see Fig. 1(a)]. Writing Kirchhoff's laws for the current and the voltage drop between points z and $z + dz$,

$$\begin{aligned} V(z, \omega) - V(z + dz, \omega) &= Z_S(\omega)dzI(z, \omega) \\ I(z, \omega) - I(z + dz, \omega) &= Y_G(\omega)dzV(z + dz, \omega), \end{aligned} \quad (4)$$

we can derive the wave equation

$$\frac{d^2 I(z, \omega)}{dz^2} = -\frac{\omega^2}{v_p^2} I(z, \omega), \quad (5)$$

where one can identify the frequency-dependant phase velocity

$$v_p(\omega) = i \frac{\omega}{\sqrt{Z_S(\omega)Y_G(\omega)}}, \quad (6)$$

which determines the TL dispersion $k(\omega) = \omega/v_p(\omega)$, where $k(\omega)$ is the wave vector of propagating wave with frequency ω . Indeed, for simple lossless TL at Fig. 1(b), we obtain $Z_S dz = i\omega L$ and $Y_G dz = i\omega C$, giving the well-known result for the constant phase velocity $v_p = dz/\sqrt{LC}$.

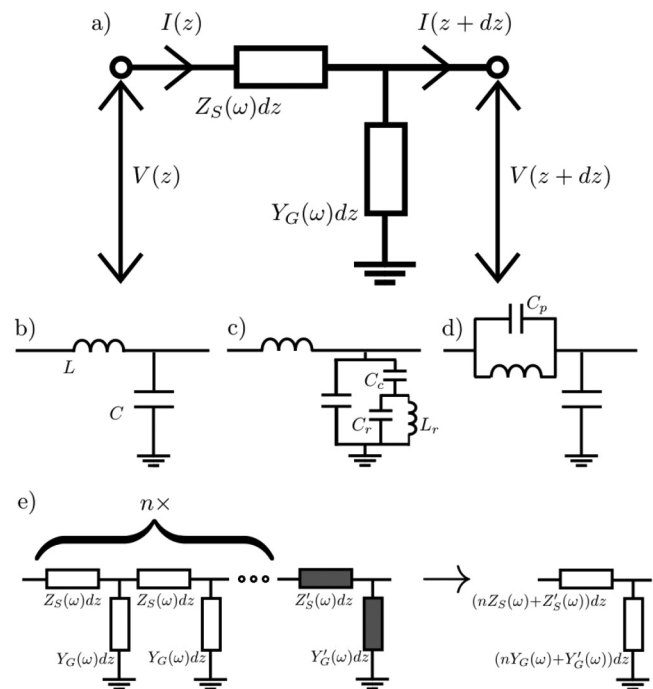


FIG. 1. Scheme of unit cell of (a) resonator embedded in transmission line; (b) simple lossless TL; (c) LC resonator; (d) LC resonator embedded in series to transmission line; (e) resonator every nth element.

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To modify the dispersion $k(\omega)$, various resonant elements can be introduced via the frequency dependence of the parameters $Z_S(\omega)$ and $Y_G(\omega)$. Let us start with an LC-resonator proposed in Refs. 19 and 30 as depicted in Fig. 1(c). The resulting dispersion relation of such TL,

$$k(\omega) = \omega \frac{\sqrt{LC}}{dz} \sqrt{1 + \frac{C_c}{C} \frac{1 - \omega^2/\omega_r^2}{1 - \omega^2/\omega_0^2}}, \quad (7)$$

results in a phase shift near the frequency $\omega_0 = \omega_r \omega_c / \sqrt{\omega_r^2 + \omega_c^2}$, where $\omega_r = 1/\sqrt{L_r C_r}$ and $\omega_c = 1/\sqrt{L_r C_c}$. This phase shift was utilized as the so-called resonant phase-matching in many recent TWPA designs.^{6,15,19–22}

Alternatively, the resonant element can be incorporated into the series impedance $Z_S(\omega)$. The simplest way is to connect a capacitor in parallel to the TL inductance as in Fig. 1(d). This creates a phase shift across the whole bandwidth and is divergent at the resonant frequency,

$$k(\omega) = \omega \frac{\sqrt{L_j C}}{dz} \sqrt{\frac{1}{1 - \omega^2/\omega_p^2}}, \quad (8)$$

where $\omega_p = 1/\sqrt{L_j C_j}$ is the plasma frequency of a single junction. Above ω_p , the wave vector is purely imaginary. The characteristic impedance of the waveguide defined as

$$Z = \sqrt{\frac{Z_S}{Y_G}} \quad (9)$$

determines the reflection coefficient $\Gamma = (Z_L - Z)/(Z_L + Z)$, which became unity above ω_p . Here, Z_L is the impedance of the input lines, typically 50Ω . The reflection of all tones above the plasma frequency avoids higher harmonic generation.

For our amplifier to operate in the frequency range from 2 to 7 GHz in 3-wave mixing, the pump (f_p) has to be set at about 10 GHz. Therefore, to suppress higher harmonics of the pump and to achieve phase matching, the frequency gap should be below 20 GHz. Thus, we choose critical current I_c and the parallel capacitance C_p such that $1/2\pi\sqrt{L_j C_p} = \sqrt{I_c/2\pi\Phi_0 C_p} \approx 20$ GHz. Since the capacitance $C_p \approx 400$ fF is determined by the size of the capacitor that is the largest element of the TWPA, it is convenient to place them equidistantly into every n th element of the TL as depicted in Fig. 1(e).

Such system can be analyzed by the introduced formalism, assuming the spacing between the resonators is small in comparison to the wavelength $ndz \ll \lambda$. Namely, it can be shown (see Appendix A) that the following model to first order in dz reduces to n -times smaller influence of the resonant phase shift. We obtained following dispersion relation:

$$k(\omega) = \omega \frac{\sqrt{L_j C}}{dz} \sqrt{\frac{n}{n+1} \sqrt{1 + \frac{1}{n} \frac{1}{1 - \omega^2/\omega_p^2}}}. \quad (10)$$

We choose $n = 5$, which leaves space for the large capacitor $C_p = 396$ fF, and the distance between them is still less than the

wavelength. For example, this capacitance could be provided by a miniature Al_2O_3 parallel capacitors. At a dielectric thickness of $d = 10$ nm, the capacitor area of $A = 20 \mu\text{m}^2$ would be sufficient. Capacitors with similar capacitance per area were fabricated using hexagonal boron nitride.³¹

The dispersion relation for the introduced design (see Fig. 1) is depicted in Fig. 2. Substituting the dispersion relation Eq. (10) to Eqs. (1)–(3), we obtained the gain profile depicted in Fig. 5. The gain is more than 15 dB with a desired flat profile from 3 to 6 GHz.

III. SIMULATION

To verify the feasibility of the proposed design and study processes not covered by the CME, we utilized the recently developed numerical simulator JoSIM^{25,26} to model JTWPA. We verified this software by comparing the results with well-known WRspice^{27,28} (see Appendix B).

The simulation setup is the same as in Ref. 14. The studied scheme is in Fig. 3 consisting of 2000 JJs. From the estimated phase velocity $v_p \sim dz/\sqrt{L_j C_g}$, where dz is the distance between JJs, the travelling time can be estimated as $t = 2000dz/v_p \approx 6.6$ ns. In order to exclude transient effects,³² we saved the input and the output currents and voltages at the first and the last node from 25 to 75 ns. We checked that our simulations are in the steady state. For example, in Ref. 14, the steady state was achieved in 10 ns. The simulation time step was set to 0.1 ps, which is significantly smaller than the one used in Ref. 14. We have verified that the results of the simulations remain consistent with smaller time steps.

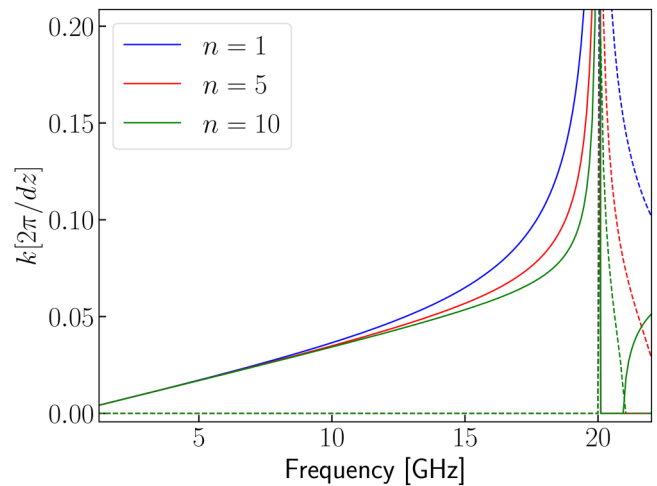


FIG. 2. Dispersion curves given by Eq. (10) for $n = 1, 5$, and 10 (blue, red, and green lines, respectively). Solid and dashed lines are real and imaginary parts of k , respectively. The wave vector is normalized to $2\pi/dz$, where dz is the distance between adjacent junctions as defined in Fig. 3. The dispersion curves diverge near plasma frequency, which provides frequency-dependent phase shift for phase matching. Their shape can be controlled by a value of n . Above the plasma frequency, the wave vector becomes imaginary, which does not allow the propagation of waves above plasma frequency.

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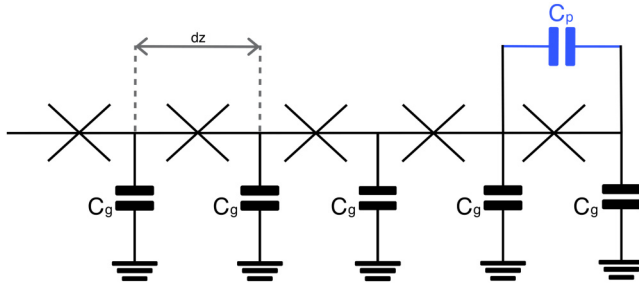


FIG. 3. Schematic of the TWPA units cell. Capacitor $C_p = 394$ fF (blue) responsible for plasma frequency placed in parallel to every fifth ($n = 5$) Josephson junction (black crosses). All Josephson junctions have the same RSCJ model: $I_c = 2 \mu\text{A}$, $R = 550 \Omega$, and $C_{JJ} = 12$ fF. All ground capacitances have equal values: $C_g = 71.5$ fF. In simulations, the whole circuit contains 400 unit cells; thus, the total amount of junction is 2000.

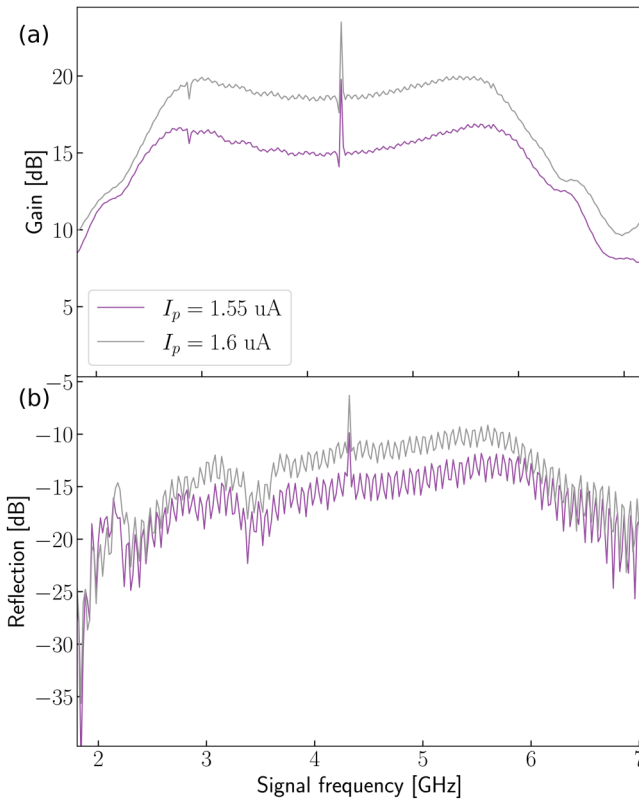


FIG. 4. Gain (a) and reflection (b) simulated in JoSIM for $I_s = 0.01 \mu\text{A}$, $I_d = 0.8 \mu\text{A}$, $f_p = 8.64$ GHz, and two values of pump amplitudes: $I_p = 1.55 \mu\text{A}$ (purple), $I_p = 1.6 \mu\text{A}$ (gray). I_p , I_s , and I_d denote the amplitudes of the current waves in the TWPA. These amplitudes are half of the values supplied by an impedance-matched current source, as described in Ref. 13. Increasing the pump amplitude can enhance the gain, but it will also increase reflections, leading to ripples.

This way, we obtained overall gain and power in each node n .^{8,14,29} The obtained gain from the simulations (see Fig. 4) is higher than 15 dB similarly to the CME result in Fig. 5 with reflection coefficient not exceeding -10 dB in the range from 3 to 6 GHz.

The agreement with CME is a noteworthy result since the theory does not account for higher harmonics, whose presence reduces the gain.⁸ Also, the CME theory works within a constant pump approximation, whereas the pump along the simulated TWPA slightly decreases. We believe that higher harmonics do not play a significant role in performance of proposed design, as they are suppressed by the plasma cut-off, as shown in Fig. 2 and demonstrated in Fig. 6. Additionally, along with the high gain, the gain ripples are within approximately 0.1 dB, which is negligible compared to, for instance, simulations of designs with periodic variations in the ground capacitance.¹⁴ The signal and idler tones increase in an almost exponential manner with the devices length up to 1000 Josephson junctions (see Fig. 6). This suggests that up to this length, phase matching is achieved [see Eq. (1)]. Then, phase mismatch develops, and the optimal length is reached at the 1500 Josephson junction. There are several reasons for phase mismatch. For example, the phase mismatch depends on the pump amplitude given in Eq. (3). The pump amplitude decreases as its energy is transferred to other harmonics (see Fig. 6). Therefore, the phase-matching conditions changes with increasing amplifier length.

Additionally to phase matching, the transmission cutoff at the plasma frequency prevents the propagation of the pump's second harmonic. The oscillations at the frequency $2f_p$ are indeed generated, but they do not propagate. The comparison to the homogeneous case, i.e., without any dispersion engineering, is in Fig. 6. Note that amplifiers with bandgaps produced by periodic modulation of their parameters^{9,14,33} exhibit the same feature. Indeed, the calculation shows that a low plasma frequency creates a cutoff that

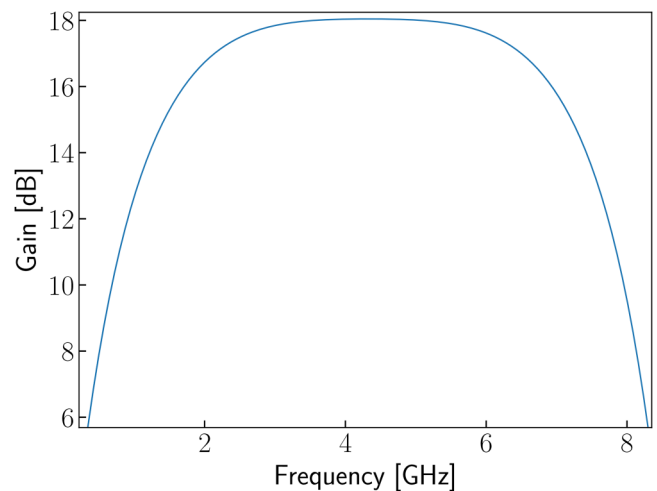


FIG. 5. Gain profile obtained by CME [see Eqs. (1), (2), (3), and (10)] for $I_d = 0.4I_c$, $I_p \approx 0.4I_c$, $f_p = 8.64$ GHz, and $I_c = 2 \mu\text{A}$. Here, I_d , I_p , f_p , and I_c are amplitudes of dc current, pump current, pump frequency, and critical current, respectively.

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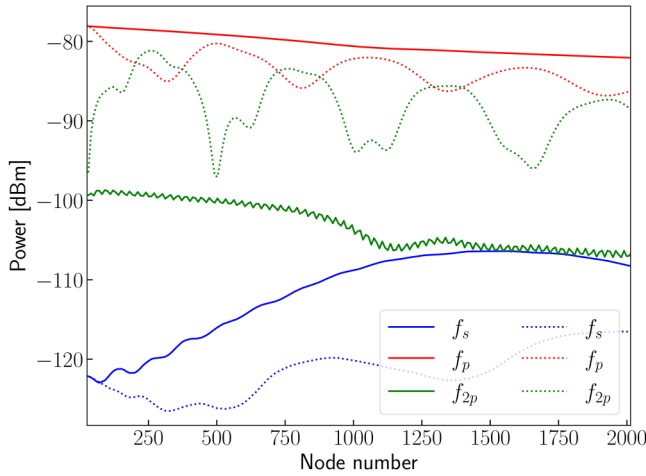


FIG. 6. Power flow of propagating plane waves for main harmonics with plasma cutoff (solid lines) and without (dotted lines). Parameters of the simulation are $f_s = 5$ GHz, $f_p = 8.64$ GHz, $I_p = 1.6 \mu\text{A}$, $I_s = 0.01 \mu\text{A}$, and $I_d = 0.8 \mu\text{A}$. Without plasma cut-off, it's clear that f_{2p} harmonic took some power from the pump.

plays similar role as a bandgap, and frequencies above 17.5 GHz are completely reflected (see Fig. 2). Lower-frequency harmonics are less reflected because they are farther from the frequency cutoff.

IV. CONCLUSION

To conclude, we have introduced a new TWPA design with phase matching provided by resonators created by the Josephson junction shunted with an additional capacitor. Moreover, this design ensures the attenuation of higher harmonics. Previous works have created several bandgaps using photonic crystals⁹ or by periodically modulating parameters.^{14,15,33} Here, we demonstrated that creating just one bandgap is sufficient, resulting in simpler amplifier design. It only requires to increase the capacitance of every fifth junction, which is feasible to fabricate.

The modified theory of TL with resonant elements has been developed to calculate the dispersion relation when a resonator is placed at every n th node. This theory was validated using simulations in JoSIM and WRspice software. Despite the amplifier showing excellent results in both programs, the results were slightly different.

ACKNOWLEDGMENTS

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AUTHOR DECLARATIONS

Conflict of Interest

The authors have no conflicts to disclose.

Author Contributions

E. Rizvanov: Investigation (equal); Writing – original draft (equal); Writing – review & editing (equal). **S. Kern:** Investigation (equal); Writing – original draft (equal); Writing – review & editing (equal). **P. Neilinger:** Investigation (equal); Writing – original draft (equal); Writing – review & editing (equal). **M. Grajcar:** Investigation (equal); Writing – original draft (equal); Writing – review & editing (equal).

DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding author upon reasonable request.

APPENDIX A: MODEL FOR PERIODIC MODIFICATIONS TO TL

Rewriting Eq. (4) in the matrix form to the first order in dz yields

$$\begin{pmatrix} V(z+dz) \\ I(z+dz) \end{pmatrix} = M \begin{pmatrix} V(z) \\ I(z) \end{pmatrix}, \quad (\text{A1})$$

where

$$M = \begin{pmatrix} 1 & -Z_S dz \\ -Y_G dz & 1 \end{pmatrix} \quad (\text{A2})$$

is the inverse ABCD matrix for the single TL element. In Figs. 1(e) and 1(a), different elements [with series impedance $Z'_S(\omega)$ and ground admittance $Y'_G(\omega)$] are inserted after every sequence of n identical elements [$Z'_S(\omega)$, $Y'_G(\omega)$]. This cascade can be described by the following inverse ABCD matrix:

$$\begin{pmatrix} 1 & -Z_S dz \\ -Y_G dz & 1 \end{pmatrix}^n \begin{pmatrix} 1 & -Z'_S dz \\ -Y'_G dz & 1 \end{pmatrix} \quad (\text{A3})$$

The n th power of the “not-primed” matrix M can be calculated by diagonalization. The n th power of eigenvalues of matrix M is to the first order in dz ,

$$\lambda_{\pm}^n = (1 \pm \sqrt{ZY} dz)^n \approx 1 \pm n\sqrt{ZY} dz, \quad (\text{A4})$$

which, together with the “primed” matrix, yields the final matrix of $n+1$ elements,

$$M_{n+1} \begin{pmatrix} 1 & -(nZ_S + Z'_S) dz \\ -(nY_G + Y'_G) dz & 1 \end{pmatrix} \quad (\text{A5})$$

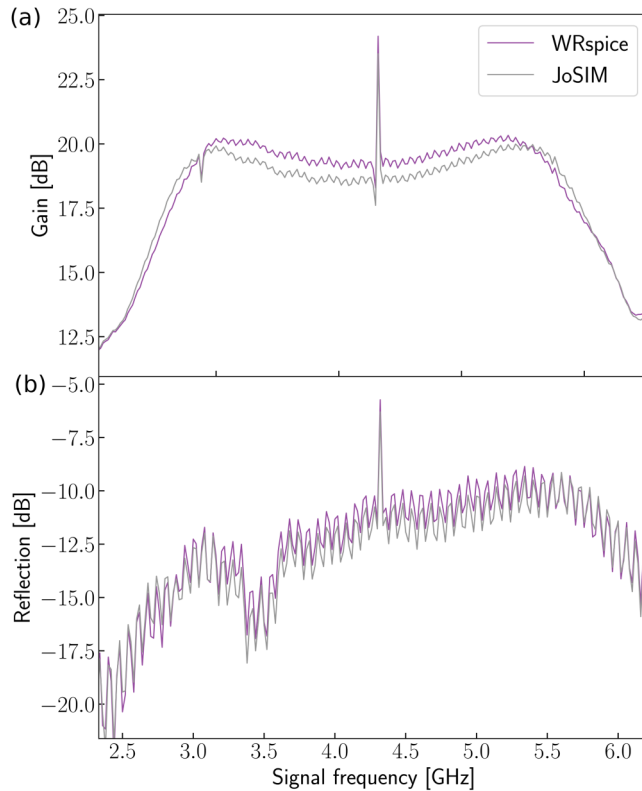


FIG. 7. Gain (a) and reflection (b) simulated in JoSIM (purple) and WRspice (gray) for $I_s = 0.01 \mu\text{A}$, $I_{dc} = 0.8 \mu\text{A}$, and $f_p = 8.64 \text{ GHz}$. Pump amplitudes are different in JoSIM ($I_p = 1.6 \mu\text{A}$) and WRspice ($I_p = 1.65 \mu\text{A}$).

The matrix M_{n+1} describes an element with series impedance $(nZ_S + Z'_S) dz$ and ground admittance $(nY_G + Y'_G) dz$, which justifies the approximation in Eq. (10).

APPENDIX B: JoSIM AND WRspice COMPARISON

At the optimal working point, the gain profile and gain amplitude in JoSIM and WRspice (maximum amplitude at reflection less than -10 dB) are approximately the same (Fig. 7). However, the WRspice requires higher pumping amplitude. JoSIM simulates noticeably faster, so we recommend using it first and then verifying the results with WRspice.

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