





RESEARCH ARTICLE | JUNE 07 2024

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O. G. Turutanov ; A. G. Sivakov ; A. S. Pokhila ; M. Grajcar 




AIP Advances 14, 065214 (2024)

<https://doi.org/10.1063/5.0211540>



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Cite as: AIP Advances 14, 065214 (2024); doi: 10.1063/5.0211540

Submitted: 2 April 2024 • Accepted: 22 May 2024 •

Published Online: 7 June 2024



O. G. Turutanov,<sup>1,2,a)</sup> A. G. Sivakov,<sup>1</sup> A. S. Pokhila,<sup>1</sup> and M. Grajcar<sup>2,3</sup>

## AFFILIATIONS

<sup>1</sup>B.Verkin Institute for Low Temperature Physics and Engineering of NAS of Ukraine, 47 Nauky Ave., 61103 Kharkiv, Ukraine

<sup>2</sup>Department of Experimental Physics, Comenius University, Mlynská dolina, 84248 Bratislava, Slovakia

<sup>3</sup>Institute of Physics, Slovak Academy of Sciences, Dúbravská cesta, Bratislava, Slovakia

<sup>a)</sup>Author to whom correspondence should be addressed: turutanov@ilt.kharkov.ua and oleh.turutanov@fmph.uniba.sk

## ABSTRACT

We prepared a bi-metal Sn/Al thin film bridge of  $1 \times 5 \mu\text{m}^2$  in size and exposed it to microwave irradiation in a frequency range of 7 to 40 GHz to explore the Shapiro steps in the current-voltage characteristics, which served as a reliable indicator for assessing current-phase relation (CPR). The measurements were made in the temperature range  $(0.89 \dots 0.99)T_c$  with  $T_c = 3.66$  K. No fractional steps are observed at 10 GHz, while all integer steps are present, and their widths oscillate with microwave field amplitude, which suggests a non-skewed quasi-sine CPR. Therefore, the normal-metal covering alters the resistive state of the long thin-film strip containing phase-slip centers so that the bi-metallic long bridge exhibits characteristics similar to a Josephson weak link. Considering a simple fabrication procedure, it may be utilized in making Josephson-effect-based devices such as DC and RF SQUIDS, especially in low-budget projects. Additional small-scale oscillations of the step widths found between the main peaks and the missing first step at a higher frequency of 20 GHz near  $T_c$  may be associated with Landau-Zener transitions between Andreev states and require further detailed study.

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## I. INTRODUCTION

The Josephson effects discovered more than 60 years ago<sup>1</sup> opened the door to the era of superconducting electronics. Many superconducting devices with Josephson junctions (JJs), such as microwave irradiation detectors, microwave mixers, voltage standards, etc., utilize the non-stationary Josephson effect. The existence of Josephson irradiation was experimentally confirmed both indirectly, as microwave-induced “Shapiro steps” in current-voltage characteristics,<sup>2</sup> and directly by measuring Josephson irradiation.<sup>3,4</sup> The generation of Shapiro steps primarily involves the current-phase relation (CPR), which is simplest for the tunnel Josephson junction (JJ) with critical current  $I_c$  and defines the supercurrent  $I_s$  as a function of the phase difference  $\varphi$ ,  $I_s = I_c \sin \varphi$ . In this case, the width of the  $n$ th current step in current-voltage curves (IVCs), denoted as  $\Delta I_n(U)$ , which appears under microwave irradiation at the dc voltage  $V_n = nhf/2e$  across the junction (where  $h$  is the Planck constant,  $f$  is the microwave frequency,  $e$  is the electron charge, and  $n$  is an integer), can be expressed by the following simple formula:

$$\Delta I_n(U) = I_c \left| J_n \left( \frac{nU}{V_n} \right) \right|, \quad (1)$$

where  $I_c$  is the junction critical current,  $U$  is the microwave field amplitude, and  $J_n$  is the Bessel function of  $n$ th order. Therefore, Shapiro steps serve as a reliable indicator of the sine CPR or can suggest a non-sinusoidal CPR in a weak link. For example, weak links with direct conductivity exhibit a non-sine CPR,<sup>5</sup> and the CPR of thin-film constriction-type weak links (Dayem bridges<sup>6</sup>) may shift from quasi-sine near the critical temperature  $T_c$  to saw-tooth-like dependence at zero temperature.<sup>7</sup> Weak links based on high- $T_c$  materials reveal the second harmonic within CPR,<sup>8,9</sup> and both long bridges and narrow channels with phase-slip centers exhibit a multi-valued CPR.<sup>5,10</sup> In scenarios involving non-sinusoidal, skewed CPR, the structure of the Shapiro step becomes considerably more intricate, featuring fractional steps associated with subharmonics.<sup>11</sup> Furthermore, topological junctions have displayed even more exotic  $4\pi$ -periodic behavior in the absence of the first step.<sup>12–15</sup>

In most applications, the pure sine current-phase relation (CPR) is favored in superconducting electronics, such as irradiation detectors, voltage standards, and SQUIDs, for its stable operation and minimal noise. However, the fabrication of tunnel junctions with preset parameters requires precise and expensive equipment, which may not be practical for low-budget, small-scale projects. In this context, we present preliminary experimental results on a novel type of superconducting weak link: a long superconducting microbridge sheathed with a thin-film normal metal. This method is distinct from the Notarys-Mercereau bridge,<sup>16</sup> which relies on the *local* suppression of superconductivity in a thin film due to the proximity effect.<sup>17</sup>

It is established that the resistive state of long narrow superconducting channels is governed by the emergence of localized non-equilibrium regions, known as phase-slip centers (PSCs)<sup>18</sup> or phase-slip lines (PSLs)<sup>19</sup> for wider thin-film strips. According to Ref. 20, the bridges exceeding  $3.49\xi(T)$  in length no longer function as Josephson weak links and exhibit a multi-valued CPR. Yet, Shapiro steps can be observed in long superconducting channels containing PSCs<sup>18</sup> or PSLs.<sup>21</sup>

We found experimentally that a relatively long superconducting bridge, when entirely coated with a normal metal film, exhibits improved Josephson behavior.

## II. EXPERIMENT AND DISCUSSION

The sample is a tin thin film covered with a thin layer of aluminum and patterned as a narrow strip with wide banks [Fig. 1(a)]. The temperature dependence of the critical current is shown in Fig. 1(b), where it is compared to that of a pure tin thin-film strip with the same in-plane size.

The normal metal coating on the superconducting film induces qualitative changes in its properties. As anticipated, the critical temperature  $T_c$  decreases due to the proximity effect, and the critical current density experiences a dramatic reduction while its temperature dependence  $I_c(T)$  modifies [Fig. 1(b)]. The critical temperatures of both samples were determined by extrapolating the  $I_c(T)$  dependences to zero critical current. The voltage jumps, typically associated with PSCs and PSLs, are smoothed, and the hysteresis vanishes in the I–V curves if Al film is placed on top of the tin film.

However, the linear sections with excess current, corresponding to the occurrence of individual PSCs, persist. One can see that up to three PSCs can enter the bridge at this temperature as the transport current increases [Fig. 2(a), curve “-inf”].

To investigate the Josephson behavior of the bi-metallic bridge, we examined Shapiro current steps in the I–V curve of the sample under microwave irradiation. For a superconducting narrow channel, the Shapiro steps were observed earlier only at frequencies above those determined by the voltage jump corresponding to the first PSC emergence,<sup>21</sup> and the step widths did not oscillate with microwave power. This behavior corresponds to a multi-valued current-phase relation for the PSC. Additionally, fractional steps are seen in the I–V curves<sup>10,18</sup> due to non-sinusoidal CPR.

For the PSCs shunted by normal metal, the picture differs. We recorded I–V curves of the sample at different temperatures and under microwave irradiation within the 1–40 GHz frequency range. Figure 2(a) shows a series of I–V curves of the bi-metallic bridge for various powers of microwave irradiation at a 10 GHz frequency. At this relatively low temperature, the I–V curves smooth out, and the Shapiro steps become visible at any low voltage and, therefore, frequency.

At a given microwave frequency and power, the step voltages are multiples of the first step voltage [Fig. 2(a)] and are proportional to the irradiation frequency [Fig. 2(b)]. However, one can see that the step voltage increment does not follow the Josephson relation  $V = hf/2e$ , being 20% less. This somewhat unexpected deviation can be attributed to the sample's structure. The normal-metal aluminum layer was deposited onto the tin strip in the same vacuum cycle, and the voltage-measuring leads were placed atop the aluminum film. Therefore, the voltage we measure corresponds to the electrochemical potential of the non-equilibrium quasiparticles in the superconducting film rather than to the electrochemical potential of the Cooper pair condensate near the oscillating PSC.<sup>18,22</sup> Given that the quasiparticle diffusion length in tin at this temperature exceeds the sample's length, the quasiparticles at the potential leads have not yet equilibrated with the Cooper pair condensate. Consequently, the measured voltage is lower than that determined by phase slips ( $d\phi/dt$ ) in the PSC core. Essentially, this results in a consistent scaling down of all measured voltages across the sample.

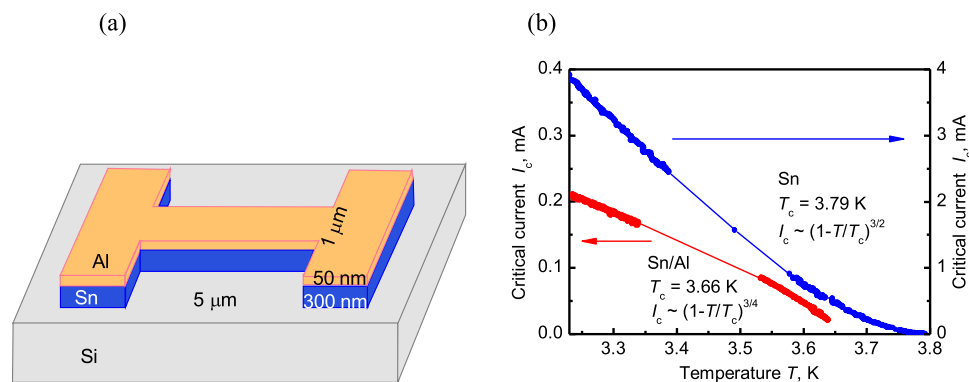
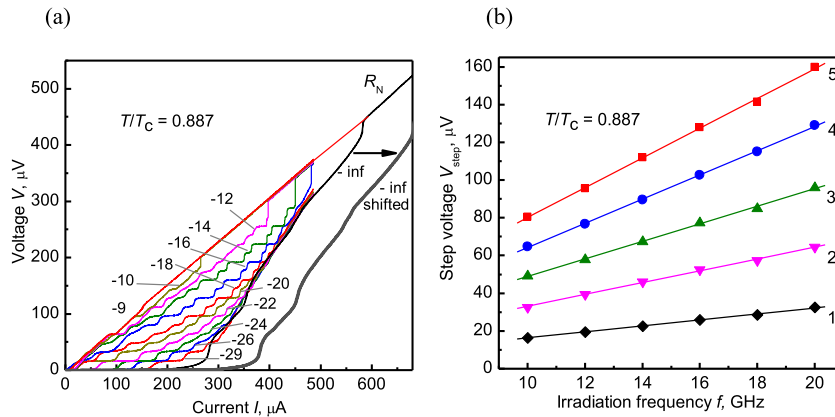


FIG. 1. (a) The scheme of the bi-metallic thin film sample and (b) critical currents of Sn and Sn/Al bridges.  $R_{4,2} = 1.63 \Omega$  (Sn);  $0.785 \Omega$  (Sn/Al).



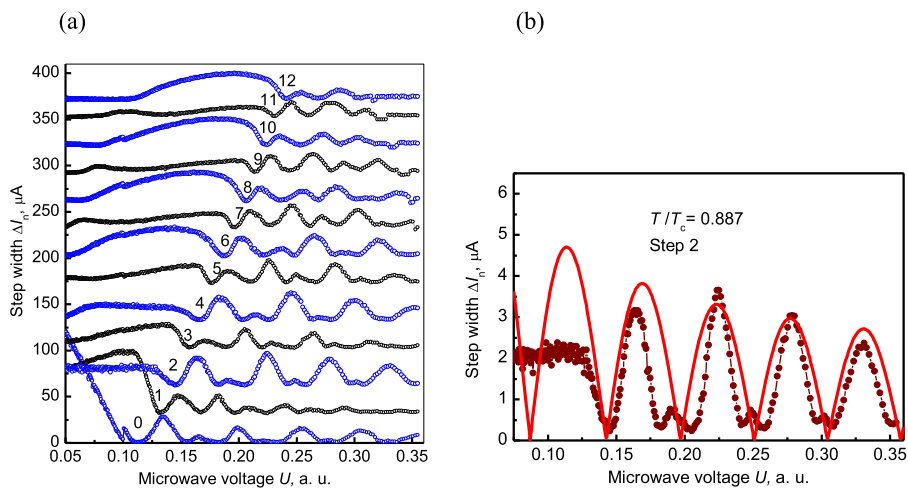
**FIG. 2.** (a) Set of I–V curves of the bi-metallic bridge for various microwave irradiation powers; the irradiation frequency is 10 GHz. The generator output microwave power is shown in dBm near each curve; “-inf” corresponds to zero power. Three PSCs are seen in the curve “-inf.” The “-inf” curve is doubled and shifted aside for clear view. (b) Voltages of the first five Shapiro steps vs microwave irradiation frequency, experimental points, and linear approximation. The step number is shown near each curve.

Considering this, the I–V curves of the sample subjected to 10-GHz microwave irradiation at various amplitudes reveal all integer Shapiro steps, as can be seen in Fig. 2(a). Unlike the “bare” superconducting bridge with PSCs,<sup>18</sup> no fractional steps are detected. Figure 3(a) illustrates the current widths of the first 12 Shapiro steps, including the critical current (zeroth step), as a function of the microwave field amplitude. The even and odd steps exhibit anti-phase oscillations with power (each curve is vertically offset to prevent overlap). The large number of distinct steps and their oscillation with microwave amplitude, along with the absence of fractional steps, suggest that the CPR is close to sine or at least not significantly skewed.<sup>11</sup> However, a detailed examination reveals deviations from the “classical” formula (1) that can be clearly seen in the behavior of, e.g., the second step [Fig. 3(b)]. The peaks are

narrower than anticipated, and their amplitude does not follow exactly the second-order Bessel function. However, the most interesting and unexpected feature is that small additional oscillations are found between the main ones.

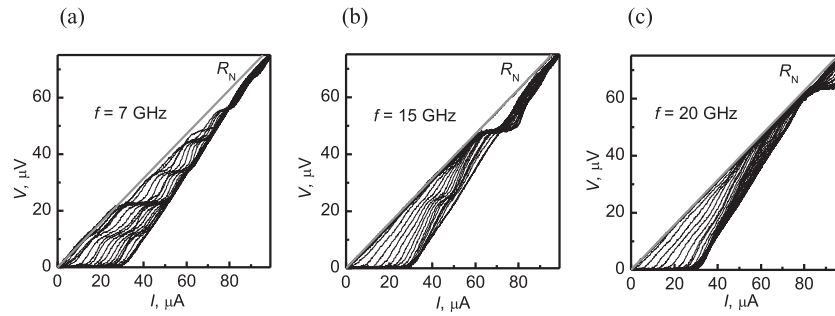
These two sets of oscillations appear to stem from two different periodic processes. They cannot be attributed to two PSCs co-existing in the bridge, as the second step voltage corresponds to the formation of a single PSC [Fig. 2(a)], and the critical current oscillations share a similar pattern. Notably, a change in the oscillation character is only seen for higher currents beyond the seventh step, likely due to the synchronized behavior of two PSCs.

The presence of additional maxima in the curve suggests parallel periodic processes: one with a  $2\pi$ -periodicity and another with a  $4\pi$ -periodicity. The differences in the amplitudes of the first even



**FIG. 3.** (a) Current widths  $\Delta I_n$  of the first 12 Shapiro steps vs 10-GHz microwave field amplitude  $U$ . Each curve is numbered and vertically shifted to avoid overlapping. (b) Step 2 oscillations (pointed curve is the experiment, solid line is the second-order Bessel function fit).  $T = 3.247$  K,  $T/T_c = 0.887$ .

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**FIG. 4.** I–V curve sets with Shapiro steps for irradiation frequency: (a) 7, (b) 15, and (c) 20 GHz. Temperature  $T = 3.63$  K,  $T/T_c = 0.992$ . Only the second step is present at the highest frequency.

and odd steps [Fig. 3(a)] support this suggestion, as they should be described by different formulas.<sup>13</sup> The  $4\pi$ -periodic CPR is typically associated with topological superconductors with a superconductor-semiconductor interface and Majorana states,<sup>12–15</sup> which is not the case for our normal metal-superconductor interface. Simple modeling of  $\Delta I_n(U)$  dependences for CPR containing  $2\pi$ - and  $4\pi$ -periodic terms shows the curve shape that differs from the experimental one, in particular, without additional zeros for smaller oscillations. Comparing the experimental curve with a Bessel function fit [Fig. 3(b)] reveals that the large peak widths could align with theoretical predictions in the absence of small oscillations. This observation suggests the idea that the hypothetical periodic processes are not independent and additive but rather competitive. From this point of view, the scenario of Landau–Zener transitions at the anti-crossing of highly transparent  $2\pi$ -periodic Andreev states, which can potentially induce  $4\pi$ -periodic behavior, as discussed in Ref. 12, may explain the additional maxima. These transitions could consequently result in the intriguing shape observed in the oscillating dependence of the Shapiro step width on the microwave field amplitude  $\Delta I_n(U)$ , considering the interplay between the non-elastic relaxation time (of order  $10^{-10}$  s in tin) and the period of the gigahertz-domain microwave irradiation. At the lowest frequencies, no Landau–Zener transitions to the excited energy level occur, and the usual  $2\pi$ -cycle behavior is observed. At higher frequencies, the  $4\pi$ -cycle manifests due to transitions to the excited energy level and a subsequent return to the ground level at the anti-crossing points. For intermediate frequencies, relaxation to the ground level can happen earlier during the slower passage along the excited energy level, contributing to the usual  $2\pi$ -cycle. This “watershed” frequency is dependent on the energy level gap at the anti-crossing point and varies with temperature, increasing as the temperature decreases. Measurements near the critical temperature confirm this assumption, as changes in the step structure at different frequencies become evident. The sets of I–V curves taken under microwave irradiation (Fig. 4) display distinct odd and even steps at 7 GHz [Fig. 4(a)], blurred odd steps at 15 GHz [Fig. 4(b)], and the absence of the first step at 20 GHz [Fig. 4(c)], which is observable at lower temperatures. These preliminary results seem compelling and require further detailed study. In this way, the Landau–Zener interferometry approach<sup>23</sup> can be a powerful tool to analyze charge transfer in Josephson junctions and weak links under microwave irradiation and explain unusual Shapiro step behavior.

### III. CONCLUSION

In summary, it can be concluded that covering a narrow superconducting tin strip with a normal metal thin film (aluminum) alters the properties of its resistive state governed by current-induced PSCs or PSLs. The resulting bi-metallic long bridge containing PSC demonstrates bright Josephson behavior, characterized by a large array of integer Shapiro steps whose width oscillates with microwave field amplitude and the absence of sub-harmonic fractional steps. These are the attributes of a non-skewed quasi-sine current-phase relation. Additionally, the bridge exhibits no hysteresis in the I–V curves, in contrast to wide tin strips<sup>21</sup> and dissipative nanobridges.<sup>24</sup> This simple one-stage fabrication procedure provides a bi-metallic weak link for constructing Josephson-effect-based devices, such as DC and RF SQUIDS, making it very useful for low-budget projects.

Additional small-scale oscillations, experimentally observed between the main peaks in the current step widths as a function of microwave field amplitude at 10 GHz, as well as a missing first step at 20 GHz, may be associated with competing  $2\pi$ - and  $4\pi$ -periodic processes due to Landau–Zener transitions between Andreev states.

### ACKNOWLEDGMENTS

The work was partially supported by the SPS Program NATO Grant No. G5796, the Slovak Research and Development Agency under the contracts Grant No. APVV-20-0425, and the Science and Technology Center in Ukraine, Project No. 9918. O.T. has been funded by the EU NextGenerationEU through the Recovery and Resilience Plan for Slovakia under Project No. 09I03-03-V01-00031.

### AUTHOR DECLARATIONS

#### Conflict of Interest

The authors have no conflicts to disclose.

#### Author Contributions

**O. G. Turutanov:** Conceptualization (equal); Data curation (supporting); Formal analysis (equal); Methodology (equal); Supervision (equal); Writing – original draft (lead). **A. G. Sivakov:** Conceptualization (equal); Data curation (lead); Formal analysis (equal);

Investigation (lead); Methodology (equal); Supervision (equal); Writing – original draft (equal). **A. S. Pokhila**: Data curation (equal); Formal analysis (equal); Investigation (equal). **M. Grajcar**: Conceptualization (equal); Formal analysis (lead); Methodology (equal); Supervision (equal); Writing – original draft (equal).

## DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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