



Stimulated emission via quantum interference: scattering of one-photon packets on an atom in a ground state

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Abstract. Usually it is assumed that the stimulated emission appears as a consequence of the Bose–Einstein statistics of photons and that to observe this effect at least two excitations have to be initially present in the atom–field system. That is, both the atom and the electromagnetic field have to be excited. In this paper we show that stimulated emission can appear exclusively as a consequence of quantum interference in a system with just a single excitation. Specifically, we consider a single two-level atom which is initially in its lower energy state and it interacts with a single-photon multi-mode wave packet. We show that for a proper choice of the photon-wave packet the atom can exhibit stimulated emission.

1. Introduction

In this paper we study a simple microscopic model describing the interaction of a single two-level atom with a *multi-mode* electromagnetic field in a perfect cavity. Our task is to study the time evolution of an atom which is initially prepared in a ground state. This atom interacts with a single-photon wave packet. We show that within this one-excitation subspace one can observe stimulated emission. Usually the stimulated emission is presented as a process when an incident light field stimulates the decay of an *excited* atom so that the effective decay rate is larger than the vacuum-decay rate given by the Fermi golden rule [1, 2]. That is, the effect is usually associated with dynamics of the atom–field system with at least two excitations.

We show that the effect of the stimulated emission can be observed also in the case when the atom is not initially excited and the electromagnetic field contains just a ‘single’ photon. We show that the time evolution of the atom depends on the initial shape of the one-photon wave packet. Specifically, we investigate the situation when the one-photon wave packet has a ‘two-peak’ structure. Intuitively speaking, the first peak of the photon-wave packet excites the atom while the second peak via quantum interference can stimulate the emission of the excited atom. We show that in the system with just a *single* excitation the effect of

stimulated emission intrinsically depends on the phase between components (peaks) of the two-peak single-photon wave packet.

2. The model

We consider a single two-level atom in a one-dimensional (1D) cavity with perfect mirrors. The interaction between the atom and the cavity field is treated within the dipole and the rotating-wave approximations neglecting all polarization and mechanical effects (i.e. the atom is considered to be a point-like particle with an infinite mass).

The free Hamiltonian for the N cavity field modes (this corresponds to the introduction of a frequency cut-off) and the two-level atom can be expressed as [1]

$$\hat{H}_0 = \hbar \sum_{n=1}^N \omega_n \hat{a}_n^\dagger \hat{a}_n + \frac{\hbar}{2} \omega_a \hat{\sigma}_z. \quad (1)$$

Here \hat{a}_n and \hat{a}_n^\dagger are the annihilation and creation operators of the n th mode, $\hat{\sigma}_z = |e\rangle\langle e| - |g\rangle\langle g|$; $|e\rangle$ and $|g\rangle$ denote the upper and lower atomic states, respectively. At the edges of the cavity, $r = 0$ and $r = L$, there are perfect mirrors, thus the space-mode functions are $g_n(k_n, r) = \sin(k_n r)$, where $k_n = \omega_n/c = n\pi/L$ (the cut-off frequency is ω_N ; c is the velocity of light). The interaction Hamiltonian in the dipole and the rotating-wave approximations (RWA) reads [1, 3]

$$\hat{H}_{\text{int}} = -\hbar \sum_{n=1}^N \lambda_n [\hat{a}_n \hat{\sigma}_+ + \hat{a}_n^\dagger \hat{\sigma}_-], \quad (2)$$

where the pseudo spin-flip operators are $\hat{\sigma}_+ = |e\rangle\langle g|$ and $\hat{\sigma}_- = |g\rangle\langle e|$. The position-dependent coupling constant λ_n is given by the expression

$$\lambda_n = \left(\frac{\omega_n}{\hbar \epsilon_0 L} \right)^{1/2} d_{eg} \sin(k_n r_a), \quad (3)$$

where d_{eg} denotes the dipole matrix element of the atom at the position r_a .

The amplitude of the electric and magnetic fields inside the cavity can be expressed as (in the Coulomb gauge, neglecting polarization)

$$\hat{E}(r) = \sum_n \left(\frac{\hbar \omega_n}{\epsilon_0 L} \right)^{1/2} (a_n + a_n^\dagger) \sin(k_n r), \quad (4)$$

$$\hat{B}(r) = i \sum_n \left(\frac{\hbar \omega_n \mu_0}{L} \right)^{1/2} (a_n - a_n^\dagger) \cos(k_n r), \quad (5)$$

where $k_n = \omega_n/c$.

The total Hamiltonian $\hat{H}_{\text{tot}} = \hat{H}_0 + \hat{H}_{\text{int}}$ describes the system of N cavity modes interacting with one two-level atom. Because of the RWA the total number of excitations in the system, $\hat{R} = \frac{1}{2}(\hat{\sigma}_z + 1) + \sum_{n=1}^N \hat{a}_n^\dagger \hat{a}_n$, is an integral of motion. The dynamics restricted to the one excitation in the system is described by the state vector (an eigenstate of \hat{R})

$$|\Psi(t)\rangle = a(t)|e\rangle_A|0\rangle_F + \sum_n b_n(t)|g\rangle_A|1_n\rangle_F. \quad (6)$$

Here $|e\rangle_A|0\rangle_F$ denotes the state vector with the atom excited, and the field modes in the vacuum; $|g\rangle_A|1_n\rangle_F$ denotes the state vector with the atom in the lower state, and the n th field mode in a one-photon Fock state and the other modes in the vacuum.

In the following we will study the time evolution of two observables, the excitation of the atom and the mean value of the energy density $I(r; t)$ of the cavity field. When the state vector of the atom–field system is given by expression (6) then $I(r; t)$ reads

$$I(r; t) = \langle : \epsilon_0 \hat{E}^2(r) : \rangle = \left| \sum_n \left(\frac{\hbar \omega_n}{L} \right)^{1/2} b_n \sin(k_n r) \right|^2, \tag{7}$$

where the colons denote the normal ordering, which is introduced in order to eliminate contributions of the vacuum.

Even though there exists the integral of motion \hat{R} in our model, it is difficult to solve the model analytically in a closed form. Therefore we will study the dynamics of our system numerically. We utilize the straightforward diagonalization of the total Hamiltonian. We assume the Hilbert space of the cavity modes can be truncated (i.e. we effectively apply the frequency cut-off) so that the Hamiltonian under consideration can be represented as a finite matrix. Then we find eigenvalues E_j and eigenvectors $|\Phi_j\rangle$ of \hat{H}_{tot} . The state vector $|\Psi(t)\rangle$ can then be written as

$$|\Psi(t)\rangle = \sum_j \exp[-iE_j(t - t_0)] |\Phi_j\rangle \langle \Phi_j | \Psi(t_0)\rangle. \tag{8}$$

This method is conceptually very simple, and in our simulations works fine.

2.1. Master equation for the atom

The atom and the field modes in an ideal cavity represent a closed system. The unitary dynamics of this system is governed by the Schrödinger equation. On the other hand, the atom represents just a subsystem of the whole atom–field system. The non-unitary time evolution of the atom is governed by the master equation for the atomic density operator in the form (for the details of how to derive this master equation see [4])

$$\frac{\partial}{\partial t} \hat{\rho} = i \frac{\delta(t)}{2} [\hat{\rho}, \hat{\sigma}_+ \hat{\sigma}_-] + \frac{\Gamma(t)}{2} [2\hat{\sigma}_- \hat{\rho} \hat{\sigma}_+ - \hat{\sigma}_+ \hat{\sigma}_- \hat{\rho} - \hat{\rho} \hat{\sigma}_+ \hat{\sigma}_-], \tag{9}$$

where the time-dependent parameter $\Gamma(t)$ and the time-dependent dynamical energy shift $\delta(t)$ can be expressed through the probability amplitude $a(t)$ defined in equation (6) as

$$\Gamma(t) = \text{Re} [\eta(t)], \quad \delta(t) = \text{Im} [\eta(t)]; \tag{10}$$

$$\eta(t) = -\frac{2}{a(t)} \frac{da(t)}{dt}.$$

In general, the parameter $\eta(t)$ can only be evaluated numerically, although analytical approximations can be derived effectively as well (see [1] and references therein). When the time-dependent parameter $\Gamma(t)$ is negative then it is related to the absorption rate of the atom in the electromagnetic field. On the other hand, if $\Gamma(t) > 0$ then it is related to the decay rate of the atom. In our work we will

consider mainly the situation when $\Gamma(t)$ is positive and therefore we will call this parameter the decay rate. From the Weisskopf–Wigner theory [5] it follows that the initially excited atom coupled to a continuum of the *vacuum* field modes in 1D free space decays exponentially to its ground state. The population of the excited atomic level P_e decays exponentially with a constant decay rate Γ_a given by the Fermi golden rule [1, 5]

$$\Gamma_a = \frac{\omega_a |d_{eg}|^2}{\epsilon_0 \hbar c}; \quad P_e(t) = \exp(-\Gamma_a t). \quad (11)$$

It can be shown (see [3]) that when in our model the atom is initially excited, then, with appropriate limits, the decay rate $\Gamma(t)$ given by equation (10) takes the value given by the Fermi golden rule.

3. Scattering of a Gaussian one-photon wave packet on an atom

In this section we will simulate propagation of a one-photon wave packet in the 1D cavity and its scattering on the two-level atom. We will show that during the process of scattering the atom acts as a semi-transparent mirror (or a beam splitter).

We consider the one-photon wave packet to be composed of one-photon number states of various cavity modes:

$$|\Psi(0)\rangle_F = \sum_{n=1}^N \exp(-ik_n r_0) f(k_n) |1_n\rangle_F. \quad (12)$$

The weight function $f(k_n)$ in equation (12) is given by the character of the field excitation in the cavity. In particular, we assume that the weight function is a discrete Gaussian function of the form

$$f(k_n) = \frac{1}{\mathcal{N}^{1/2}} \exp\left[-\frac{(k_n - k_0)^2}{4\Delta_k^2}\right], \quad (13)$$

centred in k -space around k_0 with the dispersion Δ_k ; $\mathcal{N} = (2\pi\Delta_k^2)^{1/2}$ is the corresponding normalization factor.

In the absence of the atom the one-photon wave packet (12) propagates ‘freely’ in the empty cavity[†]. Obviously, the total energy of the cavity field as well as the populations (spectrum) of the cavity modes are constant during the time evolution. The state vector $|\Psi(t)\rangle_F$ of the cavity field at time t is given by equation (12) with $r_0 \rightarrow r(t) = r_0 + ct$. It means that, except during the reflections on the mirrors[‡],

[†] The wave packet (12) with the exponential factor $-ik_n r_0$ starts to propagate from left to right; the exponential factor $+ik_n r_0$ corresponds to the propagation in the opposite direction.

[‡] For the ideal mirrors assumed here, the electric component of the field on the mirror is equal to zero. During the process of reflection the shape of the energy density is transiently changed. Its shape is ‘squeezed’ and exhibits an interference structure with pronounced maxima and minima. Within the framework of our model this reflection can be understood as a specific interference between various modes which compose the wave packet. That is, phases ϕ_n associated with different modes evolve differently (i.e. $\phi_n = \omega_n t \sim nt$), which results in a specific interference pattern which we observe as a propagation/reflection of the wave packet.

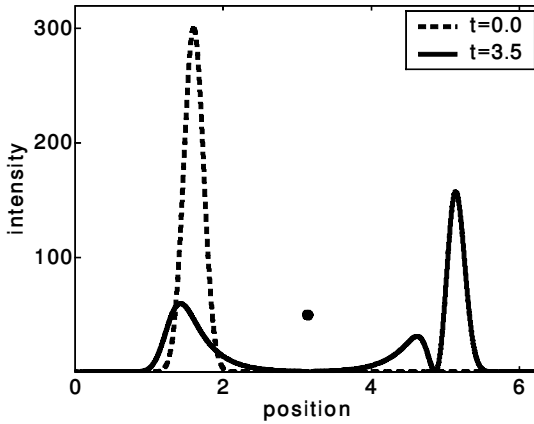


Figure 1. We present normally-ordered energy density of the Gaussian wave packet ($r_0 = 2$, $\Delta_k = 1.2\Gamma_a$) which is scattered on the atom positioned in the cavity centre ($r_a = L/2$). The energy density of the cavity field is shown at times $t = 0$ and $t = 3.5$. The other settings are $L = 2\pi$, $\Gamma_a = \pi$ and $c = 1$ in dimensionless units.

the energy density (7) preserves its shape and its peak being initially localized at the position r_0 moves with the velocity of light along $r(t) = r_0 + ct$. We note that this type of free motion of a photon wave packet and its reflection from a mirror was studied as early as the 1930s by Fermi [6]. In the presence of a single atom in the cavity the picture is changed considerably. Let us now assume that the initial wave packet (12) propagates towards the de-excited two-level atom which is positioned at the cavity centre ($r_a = L/2$).

The scattering of the Gaussian one-photon wave packet on the atom is intuitively clear. The irradiated atom partially absorbs the energy from the impinging one-photon wave packet. As soon as the atom is partially excited it starts to ‘re-emit’ the energy back to the field[†]. The scattering of the Gaussian wave packet on the atom in the cavity centre is visualized in figure 1 in terms of the energy density of the cavity field. The frequency dispersion of the wave packet is $\Delta_k = 1.2\Gamma_a$, where Γ_a is the decay rate of the atom. (In our calculations we chose units such that $\Gamma_a = \pi$.) The ‘transmitted’ part of the wave packet (travelling in the direction of the original wave packet) consists of two peaks. The first one corresponds to the attenuated original wave packet (i.e. ‘freely’ propagating part of the original wave packet) while the delayed second peak describes the re-emitted radiation in the given direction. The ‘reflected’ part of the wave packet is only due to the re-emitted radiation.

The relation between the transmitted and the reflected radiation depends mainly on the frequency of the carrier mode and the frequency dispersion of the original wave packet compared to the decay rate of the atom. Obviously, too wide a Gaussian profile in the frequency domain causes only a small fraction of the cavity modes, being close to the atomic resonance ($\pm\Gamma_a$), to interact with the atom. Consequently, just a fraction of the energy is reflected. In our particular case with

[†] The two-level atom acts as a *quantum beam splitter* or *quantum semi-transparent mirror*. After all the energy absorbed by the atom is irradiated back to the field, we may say that the energy is partially ‘transmitted’ and partially ‘reflected’.

$\Delta_k = 1.2\Gamma_a$ (see figure 1) the ratio between the transmitted and reflected energy is 61%:39%. On the other hand, more narrow (in the frequency domain) photon wave packets lead to a more efficient absorption of the incident radiation by the atom. Moreover, the absorbed radiation is preferably re-emitted in the backward direction. Namely, for $\Delta_k = 0.6\Gamma_a$ one gets the ratio 39%:61% between the transmitted and reflected energy. Making the incident wave packet effectively ‘monochromatic’ with $\Delta_k \ll \Gamma_a$, even more than 95% of the incident radiation can be reflected which is rather surprising, because in our 1D model we assume a left–right symmetry, so the originally excited atom irradiates its energy in both directions with equal probabilities (for details see [3]). What we observe in the present case is the result of quantum interference between the re-emitted light and the incident photon wave packet.

We note that due to the position dependence of the atom–field coupling constant the even modes of the cavity field are decoupled from the atom positioned in the cavity centre (at $r_a = L/2$ the mode functions $\sin(k_n r_a) = 0$ for even n). It means that only the odd modes can excite the atom. Right after the moment when the atom becomes maximally excited the envelope of the spectrum associated only with odd cavity modes has the shape of the absorption line with a minimum around the atomic resonant frequency. When all the energy absorbed by the atom is irradiated back to the field the initial spectrum of the cavity modes is recovered. Because even modes have zero coupling coefficients only half of the radiation interacts with the atom. Nevertheless, when the width of the Gaussian photon wave packet is $\Delta_k = 0.6\Gamma_a$ more than half of the radiation is reflected (and even more for smaller Δ_k). This peculiar behaviour of the energy density illustrates the intriguing features of the involved interference between various modes which form the photon wave packet. It resembles the interference effect responsible for the reflection of the wave packet on the cavity mirrors (boundaries).

4. Stimulated emission with non-Gaussian one-photon packets

Scattering of non-Gaussian one-photon wave packets on atoms is very interesting. Let us assume the one-photon wave packet to be a superposition of two mutually shifted Gaussian packets, i.e.

$$|\Psi(0)\rangle_F = \sum_{n=1}^N \mathcal{N}_\phi [\exp(-ik_n r_1) + \exp(-ik_n r_2 - i\phi)] f(k_n) |1_n\rangle_F, \quad (14)$$

where r_1, r_2 are initial positions of two peaks of the energy density (7) associated with particular Gaussian components which enter equation (14); ϕ denotes the mutual phase shift of the Gaussian components with $f(k_n)$ given by equation (13), and \mathcal{N}_ϕ is a normalization factor.

It turns out that the scattering of the wave packet (14) with the separation of the Gaussian components $|r_1 - r_2| \lesssim c/\Gamma_a$ depends very sensitively on the phase shift ϕ . In particular, let us present those values of ϕ for which the minimum and

† An analogous situation, when dynamics of a molecular system is controlled via a laser-induced quantum interference, has been discussed by Shapiro and Brumer [7]. Nevertheless, there is a fundamental difference between the two approaches—in our case we ‘control’ dynamics of the atom by a *single* photon wave packet. Shapiro and Brumer consider the control via laser (multiphoton) fields.

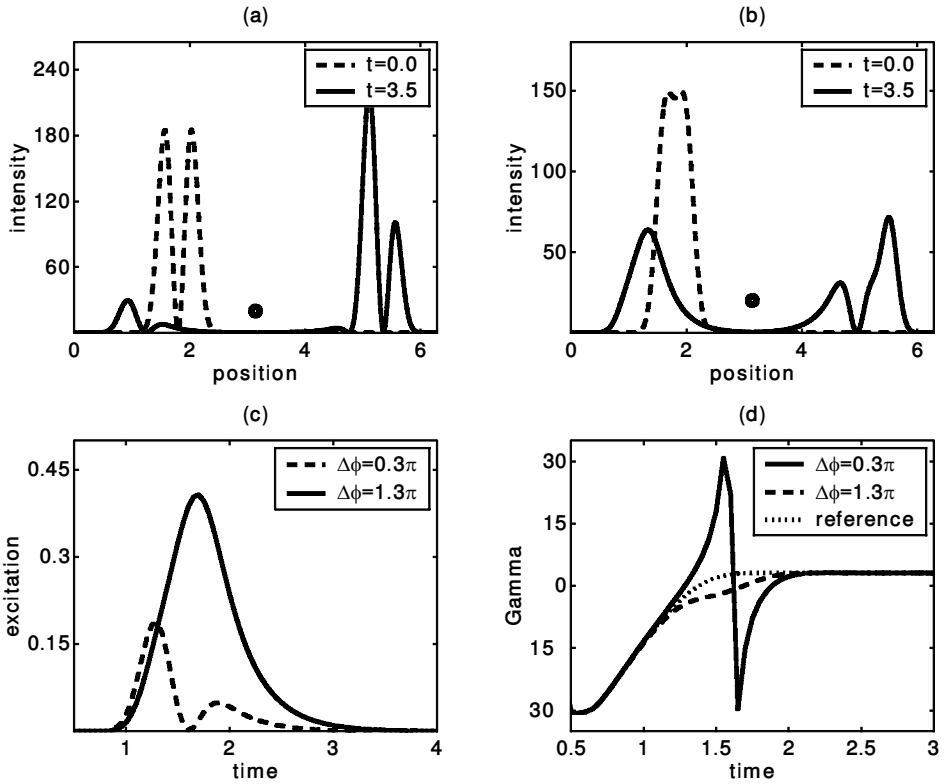


Figure 2. Scattering of the one-photon wave packet made of two Gaussian components ($r_1 = 2.0$, $r_2 = 1.6$, $\Delta_k = 1.2\Gamma_a$) on the atom positioned in the cavity centre ($r_a = L/2$). In (a) we present the energy density of the cavity field at times $t = 0$ (---) and $t = 3.5$ (—) for the one-photon wave packet (equation (14)) with the phase shift of components $\phi = 0.3\pi$. In (b) the energy density of the field mode is represented when the phase shift between the Gaussian components is $\phi = 1.3\pi$. In (c) we present the time evolution of the atomic excitation for $\phi = 0.3\pi$ (---) and $\phi = 1.3\pi$ (—). In (d) we present the time-dependent atomic decay rate $\Gamma(t)$ for $\phi = 0.3\pi$ (—) and $\phi = 1.3\pi$ (---). For reference the Gaussian photon wave packet considered in figure 1 is shown (\cdots) as well. The constant value of $\Gamma(t)$ for times $t \geq 2$ (in chosen units) is equal to $\Gamma_a = \pi$, i.e. the value given by the Fermi golden rule.

maximum excitation of the atom is achieved. In figure 2 (b) we show the scattering of the wave packet (14) with the phase shift $\phi = 1.3\pi$ of the Gaussians components ($r_1 = 2$, $r_2 = 1.6$, $\Delta_k = 1.2\Gamma_a$). The total energy density of the cavity field consists initially of just one peak, which means that owing to the interference the Gaussian components are indistinguishable. Therefore the picture for the ‘transmission’ and ‘reflection’ of the wave packet on the atom positioned in the cavity centre does not differ qualitatively from that in figure 1. Also the time-dependent atomic decay rate $\Gamma(t)$ defined by equation (10) confirms that the decay of the atom is not changed significantly, i.e. $\Gamma(t)$ is close to the ‘reference’ decay rate induced by the one-component Gaussian wave packet (12) (see figure 2 (d)). Analogously, the time evolution of the excitation of the atom (see figure 2 (c)) is essentially equivalent in both cases, i.e. the excitation probability exhibits a single peak. On the other hand,

changing the phase shift to $\phi = 0.3\pi$ a two-peak structure of the initial energy density is resolved as shown in figure 2 (a). The time evolution of the excitation of the atom in the cavity centre reflects the two-peak structure of the incident energy density. A more detailed view offers the corresponding time-dependent atomic decay rate $\Gamma(t)$ shown in figure 2 (d). Specifically, we can observe a highly enhanced decay rate associated with re-emission of radiation absorbed from the first peak in the energy density. This behaviour can be understood as a stimulated emission of the atom. It means that re-emission of the radiation absorbed from the first peak of the incident energy density is stimulated by the incident second peak of the initial energy density. This is indicated by an anomalous increase of the time-dependent decay rate $\Gamma(t)$ (solid line in figure 2 (d)). A more traditional confirmation offers the picture of the completely re-emitted radiation from the atom shown in figure 2 (a). The ‘reflected’ part of the photon wave packet is highly suppressed, which means that the atom preferably re-emits the absorbed radiation in the direction of the incident wave packet. Namely, in figure 2 (a) with $\phi = 0.3$ only 14% of the incident radiation is re-emitted backwards (‘reflected’) while 86% of the energy is propagating forwards. For comparison, in figure 2 (b) with $\phi = 1.3$ the balance between the reflected and transmitted radiation is 52% to 48%

From our results it follows that for a properly chosen phase shift between Gaussian components of the two-peak single-photon wave packet, one can observe the effect of stimulated emission with the decay rate transiently increased by an order of the magnitude. This ‘microscopic’ effect of the stimulated emission is exclusively caused by quantum interference.

To appreciate the role of quantum interference we compare the results for the one-photon wave packet prepared as the superposition of two mutually shifted Gaussian packets (14) with the corresponding mixture which is described by the initial density operator

$$\hat{\rho}(t=0)_F = \mathcal{N}_{\text{mix}}[\hat{\rho}(0; r_1)_F + \hat{\rho}(0; r_2)_F]. \quad (15)$$

The component density operators $\hat{\rho}(0; r_j)_F = |\Psi(0; r_j)\rangle_F \langle\Psi(0; r_j)|$ correspond to the state vectors given by equation (12) with $r_0 \rightarrow r_j$.

We plot the time evolution of the energy density when the initial one-photon wave packet is represented as a statistical mixture of two Gaussians in figure 3 (a). We see a striking difference between this case and the case represented in figure 2 (a) when the wave packet is in a pure superposition state. In particular, the amount of radiation reflected by the atom is much larger in the case when the wave packet is represented by a two-peak mixture. (The ratio between the reflected and transmitted radiation for the mixture (15) is the same as for the single Gaussian wave packet (12).) Moreover, in this case the decay rate (shown in figure 3 (b)) does not exhibit an increase which is typical for the stimulated emission (compare with figure 2 (d)).

It is worth noticing that the results considered in this paper also apply for scattering of photon wave packets on an ensemble of M identical two-level atoms (for references see [3]) which are confined in a small space region (compared to the resonant wavelength). Effectively, the dipole moment d_{eg} is replaced by $d_{eg}M^{1/2}$ thus appreciably increasing the strength of the atom–field interaction. It means that such an ensemble of atoms acts as a quantum semi-transparent mirror (for Gaussian packets) and its reflectivity is related to the density of atoms.

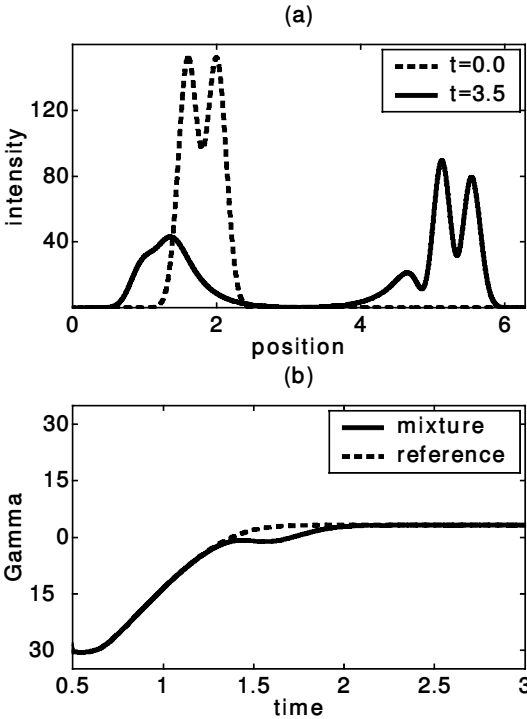


Figure 3. Scattering of the mixture of two mutually shifted Gaussian wave packets ($r_1 = 2.0, r_2 = 1.6$) on the atom in the cavity centre ($r_a = L/2$). (a) The energy density of the cavity field at $t = 0$ (---) and $t = 3.5$ (—). (b) The time-dependent atomic decay rate $\Gamma(t)$ for the mixture (—). The reference (---) corresponds to the Gaussian wave packet from figure 1.

5. Conclusions

All the effects considered in this paper have a common denominator—quantum interference of probability amplitudes. We have focused our attention on the scattering of one-photon wave packets on a single atom initially prepared in the ground state. We have shown that one atom behaves as a semi-transparent quantum mirror†. However, the balance between the ‘transmitted’ and the ‘reflected’ radiation depends mainly on shapes of incident photon wave packets. Moreover, a proper shape of the incident wave packet can enhance the re-emission rate and the stimulated emission can be observed.

Certainly there are many questions one can ask at this point. For instance, is it possible to engineer single-photon wave packets of specific forms? It is unlikely

† Let us note that the mean value of the electric field (4) in one-excitation states (6) is equal to zero. On the other hand the superposition state

$$|\Psi(0)\rangle_F = [\sum_{n=1}^N \exp(-ik_n r_0) f(k_n) |1_n\rangle_F + |0_{>F}\rangle] / 2^{1/2}$$

possesses a non-zero mean value of the electric field. This state vector describes what we could call a one-half photon wave packet (i.e. $\sum_n \langle a_n^\dagger a_n \rangle = 1/2$). The novel feature in the propagation of the one-half photon wave packet is that the phase of the mean value of the electric field in the ‘reflected’ part of the wave packet is shifted by π which (again) suggests that a single two-level atom plays the role of a quantum semi-transparent mirror.

that this would be possible with cavity fields. Nevertheless, as a matter of principle it is important to understand the microscopic nature of the stimulated emission.

Acknowledgments

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