Quantum Walk Exercises

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1. Calculate the evolution of the state $|00\cdots 0\rangle$ for a continuous-time quantum walk on a d-dimensional hypercube, with the adjacency matrix as the Hamiltonian.

2. Show that the position probability distribution of the coined quantum walk in 1D with the "balanced" coin
$$C = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & i \\ i & 1 \end{bmatrix}$$
is indeed balanced, when starting from the initial state $|x\rangle|+\rangle$. Can you find an initial state of a Hadamard walk that evolves in a balanced way?

3. Find the parameters of a unitary "Grover coin" in N dimensions.

4. Express the Ref_1 operator for a scattering quantum walk on a complete N-dimensional graph, using reflections about the states
$$|\psi_x\rangle = \frac{1}{\sqrt{N-1}} \sum_{y \neq x} |x\rangle|y\rangle.$$

5. Investigate unstructured quantum search via quantum walk – the evolution of the uniform superposition state
$$|s\rangle = \frac{1}{\sqrt{N}} \sum_{x=1}^{N} |x\rangle$$
under the Hamiltonian
$$H = -|s\rangle\langle s| - |w\rangle\langle w|,$$
where $|w\rangle$ is a marked state. How long does it take for $|s\rangle$ to evolve to something that is close to $|w\rangle$?

6. Find the eigenvectors/eigenvalues of a continuous-time quantum walk on a cycle of length L. What does the state $|0\rangle$ evolve to? Express the time-averaged limiting distribution
$$\bar{p}(x) = \lim_{T\to\infty} \bar{p}(x,T) = \lim_{T\to\infty} \frac{1}{T} \int_0^T p(x,t)\,\mathrm{d}t.$$
How large does T have to be so that $\bar{p}(x,T)$ is close to this limiting distribution?

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